Optimization of Double Wishbone Suspension using MATLAB

Rakholia Meet

Abstract— Optimization is a mathematical technique that concerns the finding of maxima or minima of functions in some feasible region Particle Swarm Optimization (PSO) is most widely used and accepted optimization technique for many of the optimization problems. In this present work, the optimization of the double wishbone suspension geometry is carried out by the above method. Two most critical parameters of the suspension geometry namely, WHEEL RATE and CAMBER GAIN are used to gain optimum SPRING ANGLE. The results obtained are fed into simulation software to see the response of the suspension system. Also the result is shown in the form of graph showing the global best and local best positions. The Wheel Rate is the force required for a unit displacement of wheel i.e. it is significant from the response point of view. The Camber Gain is related to the camber change of the wheels during turning i.e. it controls maximum tyre-road contact area during turning of the vehicle.

Index Terms— Camber Gain, Optimization, Particle Swarm Optimization (PSO), Spring angle, Wheel rate.

1 Introduction

1.1 Scope and Motivation

Double wishbone suspension geometry is the most simple and the most effective among all the geometries. It consists of two A shaped arms pivoted on the body of the roll cage which is connected to the wheel assembly. In today's competitive world and rapidly developing automobile industry optimization in each and every element associated with the automobile is very important. High level of punctiliousness is required in setting various parameters during the tuning process. Camber Gain and Wheel rate are one of the most important parameters from the point of view of ride control .The above optimization techniques are used for majority of optimization problems yielding good results. The Particle Swarm Optimization is based on the study of the behavior of the flock of the birds searching for food.This work shows the answers to following questions: What is the need of optimization? How to optimize the double wishbone suspension geometry?

The motivation is based on the need to improve the present technology to its pinnacle so as to make the maximum use of the available resources. In order to maximize the use of resources, optimization processes are widely used now-a-days. Also from academic point of view, this is a completely new branch of study thereby increasing the versatility of the knowledge.

1.2 History and Origin

The PSO algorithm was first introduced by Dr. Kennedy and Dr. Eberhart in 1995 and its basic idea was originally inspired by simulation of the social behavior of animals such as bird flocking, fish schooling and so on. It is based on the natural process of group communication to share individual knowledge when a group of birds or insects search food or migrate and so forth in a searching space, although all birds or insects do not know where the best position is. But from the nature of the social behavior, if any member can find out a desirable path to go, the rest of the members will follow quickly.

Both the researchers observed the entire phenomena. In the flock not all the birds have a good eye sight and ability to search food quickly. So after certain interval of time, a bird moves ahead with certain velocity and in certain direction depending upon the following observations:

1. A bird compares its previous position with the current position after a definite period and decides a best position depending on the closeness to the target locally.
2. A bird also compares its current position with the global best position i.e. the best postion among the entire flock and approaches towards it with a certain velocity and acceleration.

After some time, the birds converge to a single place by following the above procedure.This is the optimized result obtained by social and local interactions. This was then put in a mathematical form and was named as Particle Swarm Optimization (PSO) method.

The PSO method is becoming very popular because of its simplicity of implementation as well as ability to swiftly converge to a good solution. It does not require any gradient information of the function to be optimized and uses only primitive mathematical operators.. In addition, there are few parameters to adjust in PSO. That's why PSO is an ideal optimization problem solver in optimization problems. PSO is well suited to solve the non-linear, non-convex, continuous, discrete, integer variable type problems.

2 Background

2.1 Optimization

Optimization provides us with the best-suited solution to problem under given circumstances. Optimization refers to both minimization and maximization tasks. Since the maximization of any function is mathematically equivalent to the minimization of its additive inverse, the term minimization and optimization are used interchangeably. For this reason, now-a-days, it is very important in many professions.

Optimization problems are classified into two types:

1. Linear Optimization
2. Non linear Optimization

Based on the problem characteristics, optimization problems are classified in the following:

1. Constrained optimization
2. Unconstrained optimization
3. Dynamic optimization
4. Global optimization
5. Local optimization

2.2 Particle Swarm Optimization (PSO)

The Particle Swarm Optimization (PSO) algorithm is a multi-agent parallel search technique .In this technique in which each particle represents a potential solution in the swarm. All particles fly through a multidimensional search space where each particle is adjusting its position according to its own experience and that of neighbors. Suppose $\mathbf{x}^i_t$ denote the position vector of particle in the multidimensional search space (i.e. R^n) at time step, then the position of each particle is updated in the search space by

$$\mathbf{X}^i_{t+1} = \mathbf{X}^i_t + \mathbf{V}^i_{t+1} \quad \textbf{with} \quad \mathbf{X}^i_0 \sim \mathbf{U}(\mathbf{X}_{min}, \mathbf{X}_{max})$$

Where,

$\mathbf{V}^i_t$ is the velocity vector of particle that drives the optimization process and reflects both the own experience knowledge and the social experience knowledge from the all particles;

$\mathbf{U}(\mathbf{X}_{min}, \mathbf{X}_{max})$ is the uniform distribution where $\mathbf{X}_{min}$ and $\mathbf{X}_{max}$ are its minimum and maximum values respectively.

Therefore, in a PSO method, all particles are initiated randomly and evaluated to compute fitness together with finding the personal best (best value of each particle) and global best (best value of particle in the entire swarm). After that a loop starts to find an optimum solution. In the loop, first the particles' velocity is updated by the personal and global bests, and then each particle's position is updated by the current velocity. The loop is ended with a stopping criterion predetermined in advance

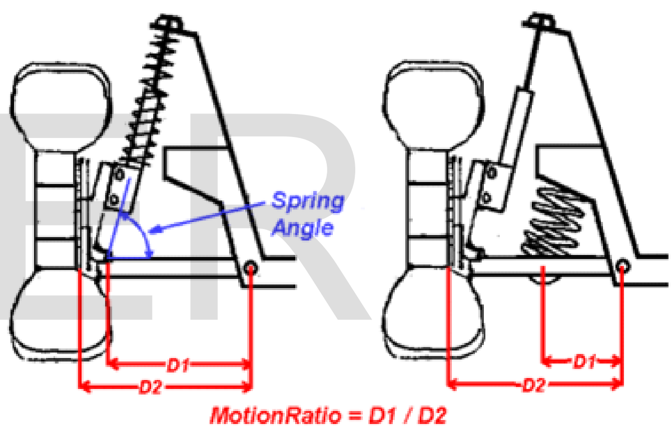
Basically, two PSO algorithms, namely the Global Best (*gbest*) and Local Best (*lbest*) PSO are calculated for carrying out the optimization.

2.3 Wheel Rate

Wheel Rate is basically the Spring Rate but measured at the wheel instead of the where the spring attaches to the linkage. The figure to the right shows how wheel rate is determined. It is basically the Spring Rate multiplied by the motion ratio squared. The motion ratio is the ratio between how much the spring is compressed compared to how much the wheel is actually moved. For example if the spring only compresses 0.6 inches when the wheel is moved 1 inch, that would be a .6 motion ratio.

This motion ration can be estimate by knowing the distance of the wheel to the pivot point of the suspension arm and the distance to the spring from the pivot point.

The "Spring Angle Correction" is typically a small correction equal to the cosine of the spring angle from vertical.



So the equation for Wheel Rate is the Motion Ratio squared times the Spring Rate.

Wheel Rate = Spring Rate * (Motion Ratio ^ 2) * Spring Angle Correction

2.4 Chamber Gain

The camber change rate is a function only of the *front view swing arm length*, fvsa length. Front view swing arm length is the length of the line from the wheel centre to the instant centre when viewed from front.

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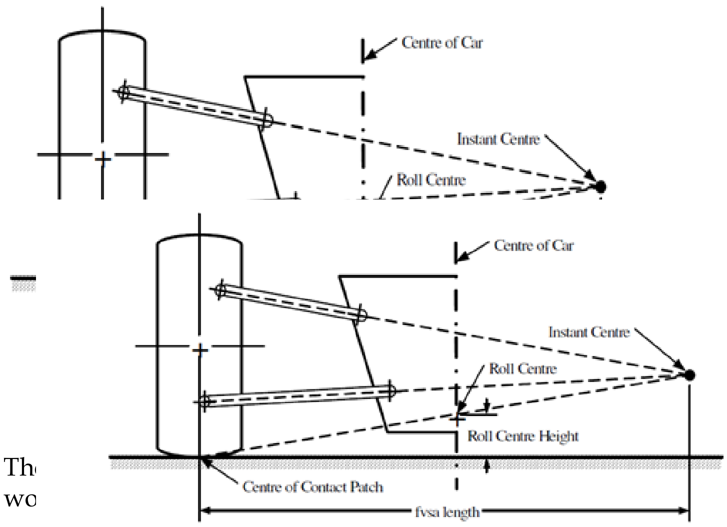


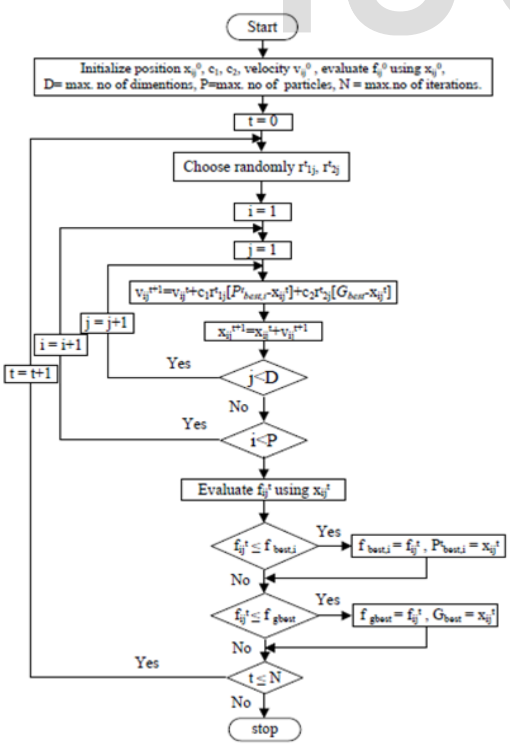
Th wo

$$degrees/mm = \arctan\left(\frac{1}{fvsa\ length}\right)$$

The camber change is not constant throughout the whole ride travel since the instant centre also moves with wheel travel.

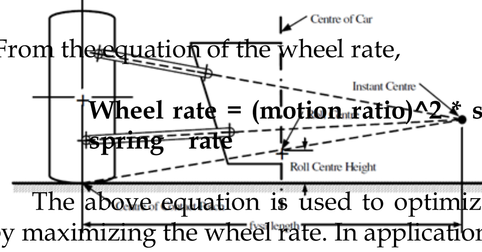
3 MATHEMATICAL MODELING

3.1 PSO Algorithm



3.2 Mathematical model of double wishbone suspension

3.2.1 Objective function of spring angle using the wheel rate

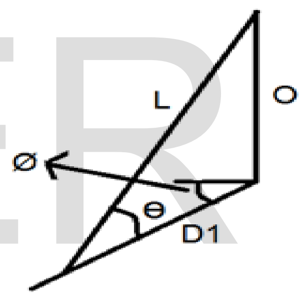


From the equation of the wheel rate,

Wheel rate = (motion ratio)^2 * sin(spring angle) * spring rate

The above equation is used to optimize the spring angle by maximizing the wheel rate. In applications related to the off –roading the wheel travel should be sufficiently high in order to easily overcome the large unevenness in the terrains. So the wheel travel must be maximum. Hence the wheel rate must be minimum.

Here, the spring rate is constant while the motion ratio is the function of the spring angle. The above equation is the function of the spring angle only. Thus, it has only one independent variable so it can be optimized using linear optimization which will be constrained by limited values of the spring angle.



Let,

Spring angle with the A- Arm= **Ө**
The distance between spring mount and pivot of A-Arm = **D1**
Distance between the upper A-arm mounting point and spring mounting point= **O**
Initial spring length when not compressed = **L**
Initial A arm angle where the spring is mounted = **Ø**

Formulation of the objective function:

From the above geometry we can write the D1 in the form of spring angle Ө using the cosine rule:

Let WR(Ө) be the objective function.

WR(Ө) =((0.278) – (sin(Ө+26)*(0.232)))* sin(Ө)………(1)

The above function is used for optimization.

Here,
O = 10 inches
L = 18.2 inches
Ø = 26 degree

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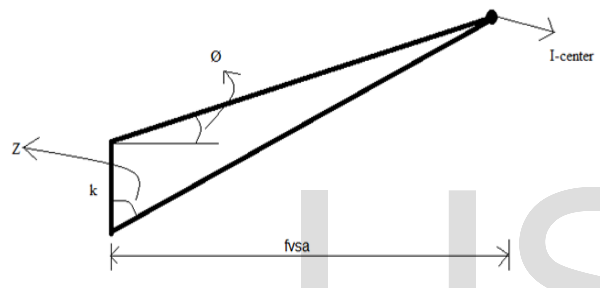
The above values are taken from the values of the ATV vehicle being manufactured in the campus.

3.2.2 Objective function of A-arm angle using chamber gain

From the equation of the camber gain,

Camber Gain = $\tan^{-1}$ (1/fvsa)

The above equation is used to optimize the A-arm initial angle by minimizing the camber gain. In applications related to the off roading, the wheel should remain straight for sufficiently long time in order to maintain zero camber condition so that the maximum contact patch is obtained. This increases the stability and tire grip during the turning of the vehicle avoiding skidding of tires and securing maximum traction in tires also during the turning.



Let,

Distance between the upper and lower knuckle mounting points = **k**
Initial A arm angle where the spring is mounted = **Ø**
The angle between lower A-Arm and the knuckle plane = **Z**

Formulation of the objective function:

Let CG be the camber gain objective function.By applying sine rule in the above geometry,

CG = $\tan^{-1}$(0.89*cot(Z)) - 0.438…………..(2)

The above function is used for optimization.

Now,
Ø = 26 degree
k = 10 inch

The above values are taken from the values of the ATV vehicle being manufactured in the campus.

4 MATLAB ANALYSIS

4.1 Analysis of Wheel Rate

The equation (1) is fed into the MATLAB code in order to optimize the spring angle.

Function declaration:

```
function out=wr(x)
out=((0.278)*sin(x))-((0.104)*(1-cos(x.*2)))-((0.05)*(sin(x.*2)));
```

Function calling for feeding into optimization code:

```
y=pso_meet('WR',1,4,[20 40],0,[100 20000 24 2 2 0.9 0.4 1500 1e-25 250 NaN 0 0],'goplotpso',0)
```

4.2 Analysis of Chamber Gain

The equation (2) is fed into the MATLAB code in order to optimize the initial lower A-arm angle.

Function declaration:

Function calling for feeding into optimization code:

```
function out=wr(x)
out=atan(0.89*cot(x*0.017))-0.438;
```

Here, **pso_meet** is the optimization code used for 1 dimen-

```
y=pso_meet('cg',1,4,[20 40],1,[100 20000 24 2 2 0.9 0.4 1500 1e-25 250 NaN 0 0],'goplotpso',0)
```

sional linear unconstrained optimization

5 RESULTS

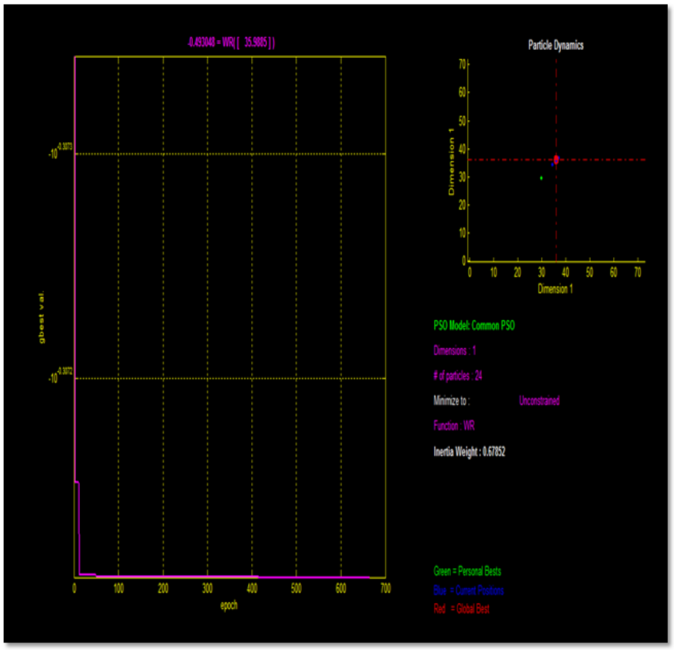
5.1 Result based on Wheel Rate

The output of the MATLAB analysis is shown below:

```
>> y=pso_meet('WR',1,4,[20 40],0,[100 20000 24 2 2 0.9 0.4 1500 1e-25 250 NaN 0 0],'goplotpso',0)
PSO: 1/20000 iterations, GBest = -0.49278370339632377.
PSO: 100/20000 iterations, GBest = -0.49304761453916696.
PSO: 200/20000 iterations, GBest = -0.49304761453916696.
PSO: 300/20000 iterations, GBest = -0.49304761460933061.
PSO: 400/20000 iterations, GBest = -0.49304761460942514.
PSO: 500/20000 iterations, GBest =  -0.4930476146094252.
PSO: 600/20000 iterations, GBest =  -0.4930476146094252.
PSO: 665/20000 iterations, GBest =  -0.4930476146094252.

--> Solution likely, GBest hasn't changed by at least 1e-025 for 250 epochs.

y =

   35.9885
   -0.4930
```

The result gives optimum value of **ϴ = 35.9885°** with the constraints in range (20, 40).

Graph of the above result is shown below:



5.2 Result based on Camber Gain

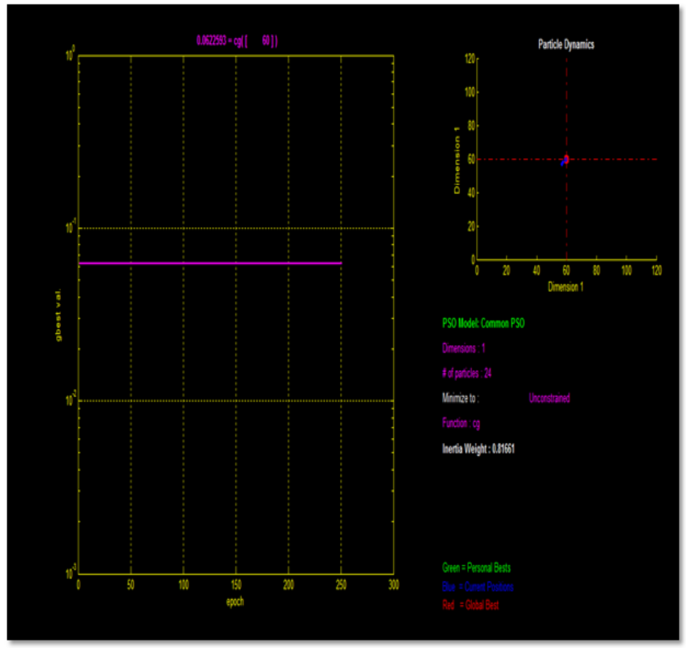
The output of the MATLAB analysis is shown below:

```
>> y=pso_meet('cg',1,4,[20 60],0,[100 20000 24 2 2 0.9 0.4 1500 1e-25 250 NaN 0 0],'goplotpso',0)
PSO: 1/20000 iterations, GBest =  0.06225932541692375.
PSO: 100/20000 iterations, GBest =  0.06225932541692375.
PSO: 200/20000 iterations, GBest =  0.06225932541692375.
PSO: 251/20000 iterations, GBest =  0.06225932541692375.

--> Solution likely, GBest hasn't changed by at least 1e-025 for 250 epochs.

y =

   60.0000
    0.0623
```

The result gives optimum value of **z = 60°** with the constraints in range (20, 40)

Graph of the above result is shown below:

5 Conclusion



The above value of the spring angle gives a wheel travel of 8 inches for a car of 250 kg and 40:60 weight transfer.These values are quite good for an All Terrain Vehicle in order to avoid rough and highly uneven terrain. Such a high value of wheel travel is good for safe and sound ride in the rough terrain.The 60 degree angle of the lower arm gives a very low camber gain of 0.513degree/inch. If we divide the total travel in two parts 4 inches upwards and 4 inches downwards then we get a total camber change of maximum 2 degrees upwards and downwards which is permissible. Hence, the optimized values are reliable and can be used in the practical purposes.

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